# GRAPH THEORY 

## Tutorial - 3

1) For $n \geq 4$, prove that the minimum number of edges in an n-vertex graph with diameter 2 and maximum degree $(n-2)$ is $(2 n-4)$.
2) Prove or disprove: Every tree has at most one perfect matching.
3) Prove that every maximal matching in a graph $G$ has at least $\alpha^{\prime}(G) / 2$ edges.
4) Suppose that $G$ is a graph and $D$ is an orientation of $G$ that is strongly connected. Prove that if G has an odd cycle, then D has an odd cycle.
